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Laboratoire de Recherche Opérationnelle, Combinatoire, Informatique Théorique et Méthodes Stochastiques – RECITS

Recueil des Résumés

Journées Scientifiques du Laboratoire

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Alger, Algérie
21 - 22 Avril 2014

Objectifs :

Ces journées scientifiques sont une opportunité offerte aux chercheurs du laboratoire, et principalement les doctorants, pour communiquer, échanger et confronter leurs résultats avec leurs confrères du même domaine de recherche. Elles se veulent une rencontre qui permettra de stimuler et de créer un cadre favorable pour le développement et la promotion de leurs recherches et de leurs idées.

Thèmes des journées (Liste non limitative) :

- Recherche Opérationnelle
- Combinatoire
- Informatique Théorique
- Méthodes Stochastiques

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Présentation du Laboratoire RECITS
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RECITS est un laboratoire de recherche opérationnelle, de combinatoire, d'informatique théorique et de méthodes stochastiques agréé par l'arrêté ministériel n° 242 du 03 avril 2013. Il est localisé à la Faculté de Mathématiques de l'Université des Sciences et de la Technologie Houari Boumediene – Alger. Il est dirigé par le Professeur BOUDHAR Mourad

Le Laboratoire compte 54 personnes dont : 2 Professeurs, 5 Maîtres de Conférences A, 5 Maîtres de Conférences B, 8 Maîtres Assistants A, 9 Maîtres Assistants B et 23 Doctorants LMD. 18 des maîtres assistants et attachés de recherche préparent une thèse de Doctorat.

Le laboratoire est structuré en cinq équipes de recherche :

CATI : Combinatoire, Arithmétique et Informatique Théorique
RO-TOP : Recherche Opérationnelle pour le Transport et l'Ordonnancement en Productique
CEAPA : Combinatoire Enumérative, Analytique, Probabiliste et Applications
STEP : Séries Temporelles, Econométrie et Probabilités
O2M : Optimisation à Objectifs Multiples

Dans quelles revues publier ? et analyse scientométrique de la production Algérienne en mathématique

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Résumé :

Nous donnerons un panorama des revues et des gros éditeurs dans le monde de la production scientifique et présenterons quelques indicateurs les concernant. Dans une seconde phase, nous focaliserons sur la production en mathématiques avec quelques statistiques récentes.

Time Frequency Array Signal Processing: A synergistic relationship between time frequency methods and sensor array processing

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Abstract: Conventional time-frequency analysis methods were only recently extended to data arrays, and there is the potential for a great synergistic development of new advanced tools by exploiting the joint properties of time-frequency methods and array signal processing methods. Conventional array signal processing assumes stationary signals and mainly employs the covariance matrix of the data array. This assumption is motivated by the crucial need in practice for estimating sample statistics by resorting to temporal averaging under the additional hypothesis of ergodic signals. When the frequency content of the measured signals is time varying (i.e., nonstationary signals), this class of approaches can still be applied. However, the achievable performances in this case are reduced with respect to those that would be achieved in a stationary environment. Instead of considering the nonstationarity as a shortcoming, time frequency and array processing (TFAP) takes advantage of the nonstationarity by considering it as a source of information in the design of efficient algorithms in such nonstationary environments. This talk deals with this synergistic relationship between time-frequency methods and array signal processing methods.

The speaker plans to address a broad audience with general background in mathematics.

Biography speaker: Adel Belouchrani was born in Algiers, Algeria, on May 5, 1967. He received the State Engineering degree in 1991 from Ecole Nationale Polytechnique (ENP), Algiers, Algeria, the M.S. degree in signal processing from the Institut National Polytechnique de Grenoble (INPG), France, in 1992, and the Ph.D. degree in signal and image processing from Télécom Paris (ENST), France, in 1995. He was a Visiting Scholar at the Electrical Engineering and Computer Sciences Department, University of California, Berkeley, from 1995 to 1996. He was with the Department of Electrical and Computer Engineering, Villanova University, Villanova, PA, as a Research Associate from 1996 to 1997. From 1998 to 2005, he has been with the Electrical Engineering Department of ENP as Associate Professor. He is currently and since 2006 Full Professor at ENP. His research interests are in statistical signal processing, (blind) array signal processing, time-frequency analysis and time-frequency array signal processing with applications in biomedical and communications. He is currently Associated Editor of the IEEE Transactions on Signal Processing.

Dénombrabilité et non dénombrabilité

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Résumé :

Nous parlerons des notions d'infini, de dénombrabilité et de non dénombrabilité. On aborde ensuite quelques notions de base sur la théorie des ensembles pour terminer avec l'hypothèse du continu.

Multiobjective Quadratic Integer Programming and Applications

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In optimization, it is widely known that linear programming is a powerful tool to solve many real world applications. Nevertheless, models of the real world are not always of a linear form; de facto, in part they can be expressed as nonlinear. In addition to that, most everyday decisions are made on the basis of conflicting objectives that we want to be as good as possible. Thus, and contrary to the case of single objective, finding the optimal solution is no longer the goal but a set of best compromise solutions instead. In this study we deal with nonlinear multiobjective optimization and propose through it an exact method to solve the Multiobjective Quadratic Integer Linear Programming Problem (MQILP).

We first conducted an extensive literature review, looking for other studies related to our topic but to no avail. There is only the Multiobjective Quadratic Assignment Problem that has been studied using meta-heuristics [1] and no exact method exists till now for the MQILP. However, the most reported applications with only one objective were the Quadratic Assignment Problem [2] and the Quadratic Knapsack Problem [3]. Also, we have noticed in finance in the presence of continuous variables, the famous Mean-Variance Markowitz's Model for the portfolio selection problem ([4], [5]).

In this presentation, a generalization of our method in the linear case [6] is described to generate the entire set of efficient solutions for MQILP. Based on Branch & Cut technique and Taylor Series Approximation of the objective functions, our method requires only a single optimization per iteration and explores the feasible domain smartly. This is made possible by using the increasing directions of the quadratic functions given by the corresponding gradients, what allows us to delete considerably non efficient solutions without have to compute them.

Applications to the Portfolio Selection Problems are done. In the 1950s, Markowitz [4] established the modern portfolio theory and proposed a model selection procedure of several assets, based on statistical criteria, in order to obtain optimal portfolios. Markowitz showed that the investor seeks to optimize its choice based on risk/return relationships. Mostly, the "efficient portfolio" is determined for a given level of return. Whereas, considering simultaneously the conflicting objective functions is more judicious. We propose to adapt our general method to solve a modified version of the classical Mean-Variance (MV) portfolio selection model; that is the Limited Asset Markowitz (LAM) model, noted (P). Indeed, the MV model was modified in the literature by several authors by imposing new constraints on the number of assets to be included and their weights in the portfolio. Also, due to integer aspect of the decision variables, this problem falls into the class of NP-hard problems and its resolution can be made by using our method.

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Preserving log-convexity for s-Pascal triangle

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Abstract

Preserving log-convexity property is established for the generalized Pascal triangles. We generalize a result of H. Davenport and G. Pólya: "On the product of two power series", where they proof that the ordinary Pascal triangle satisfy the preserving log-convexity property.

Key words : log-convexity, linear transformations, ordinary multinomials.

Let $s \geq 1$ and $n \geq 0$ be two integers, and $k = 0, 1, ..., sn$, the element $\binom{n}{k}_s$ of generalized Pascal triangle is defined as the k^{th} coefficient in the development

$$(1 + x + x^2 + \cdots + x^s)^n = \sum_{k \geq 0} \binom{n}{k}_s x^k. \tag{0.1}$$

A sequence of nonnegative numbers $\{x_k\}_k$ is log-convex (LX for short) if $x_{i-1}x_{i+1} \geq x_i^2$ for all $i > 0$. This is equivalent to $x_{i-1}x_{j+1} \geq x_i x_j$ for all $j \geq i \geq 1$.

Let us consider the following two linear transformations of sequences

$$z_n = \sum_{k=0}^{ns} \binom{n}{k}_s x_k, \quad (n \geq 0), \tag{0.2}$$

and

$$t_n = \sum_{k=0}^{ns} \binom{n}{k}_s x_k y_{ns-k}, \quad (n \geq 0). \tag{0.3}$$

1. We say that the linear transformation (0.2) has the preserving log-convexity property (by short PLX) if it preserves the log-convexity of sequences, i.e. the log-convexity of $\{x_n\}$ implies that of $\{z_n\}$
2. We say that the linear transformation (0.3) has the double PLX property if it preserves the log-convexity of sequences, i.e. the log-convexity of $\{x_n\}$ and $\{y_n\}$ implies that of $\{t_n\}$.

We establish our main result.

Theorem 0.1. *We have the following*

1. *If the sequence of nonnegative numbers $\{x_k\}_k$ is log-convex, then so is the sequence $\{z_k\}_k$.*
2. *If the sequences of nonnegative numbers $\{x_k\}_k$ and $\{y_k\}_k$ are log-convex, then so is the sequence $\{t_k\}_k$.*

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Etude comparative entre AG et PSO appliquées au Problème d'Ordonnancement avec Blocage

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Résumé : Nous présentons dans cet article une étude comparative de deux méta-heuristiques, connues dans la littérature pour leur efficacité, les Algorithmes Génétiques AG et l'Optimisation par l'Essaim de Particules PSO. Ces deux méta-heuristiques ont été testées sur le problème d'ordonnancement avec blocage connu dans la littérature pour être NP-Difficile. Le fait que les machines n'admettent pas d'espace de stockage, complique davantage le problème. Le but de notre travail est d'étudier l'efficacité de ces deux méta-heuristiques AG et PSO en considérant cette contrainte, dite de blocage.

Mots clés : Ordonnancement – Job shop – Blocage – Algorithmes génétiques – PSO – Métaheuristiques.

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Reentrant flow shop with an exact time lag

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Abstract : This paper focuses on a chain-reentrant flow shop with two machines and exact time lag L, in which each task may be processed in this order M_1, M_2, M_1 and there is an identical time lag between the completion time of the first operation and the start time of the second operation on the first machine. The objective is to minimize the total completion time. As the problem is proved to be NP-Hard, we present an integer program to solve small instances, we propose some heuristics to solve the general problem and we give a subproblem that can be solved in polynomial time based on the maximum weight matching problem.

Keywords : flow shop, time lags, reentrance, makespan, complexity, heuristics.

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Recurrence relations and combinatorial identities for r-Lah numbers

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Abstract: We give a new combinatorial interpretation for r-Lah numbers. We also express r-Lah numbers in terms of Lah numbers. Finally, we give an application related to rising and falling factorial powers.

Keywords: Lah numbers, r-Lah numbers.

1 Introduction

For positive integers n, k, r the r-Lah numbers, see for instance [2], count the number of partitions of the set $1, 2, \ldots, n$ into k ordered lists with the restriction that the elements $1, 2, \ldots, r$ belong to distinct lists. Note that as for Lah numbers, the r-Lah numbers satisfy for $n > r$, the recurrence relation

$$\left\lfloor \begin{matrix} n \\ k \end{matrix} \right\rfloor_r = \left\lfloor \begin{matrix} n-1 \\ k-1 \end{matrix} \right\rfloor_r + (n+k-1) \left\lfloor \begin{matrix} n-1 \\ k \end{matrix} \right\rfloor_r . \tag{1}$$

with $\left\lfloor \begin{smallmatrix} n \\ k \end{smallmatrix} \right\rfloor_r = 1$ if $n = k = r$ and $\left\lfloor \begin{smallmatrix} n \\ k \end{smallmatrix} \right\rfloor_r = 0$ if $n < r$.

Our motivation is to investigate combinatorial approach to give two cross recurrence relations for r-Lah and some propreties.

2 Main results

The relation (1) gives a recurrence relation with fixed r. Bellow, we establish a cross recurrence relation with respect to r.

Theorem 1 [1] *The r-Lah numbers satisfy the recurrence relation*

$$\left\lfloor \begin{matrix} n \\ k \end{matrix} \right\rfloor_r = (k-r+1)\frac{(n+r-2)}{(k+r-1)} \left\lfloor \begin{matrix} n-1 \\ k \end{matrix} \right\rfloor_{r-1} + \frac{(n+r-2)}{(k+r-2)} \left\lfloor \begin{matrix} n-1 \\ k-1 \end{matrix} \right\rfloor_{r-1} . \tag{2}$$

The following theorem gives a cross recurrence relation for $\left\lfloor \begin{smallmatrix} n \\ k \end{smallmatrix} \right\rfloor_r$. The main difference with relation (2) is that the coefficients appearing in the recurrence relation are integers.

Theorem 2 [*1*] *For any nonnegative integers* $0 \leq r \leq k \leq n$*, we have*

$$\left\lfloor \begin{matrix} n \\ k \end{matrix} \right\rfloor_r = \sum_{i=0}^{n-k} (i+1)! \binom{n-r}{i} \left\lfloor \begin{matrix} n-i-1 \\ k-1 \end{matrix} \right\rfloor_{r-1}. \tag{3}$$

As an application of the r-Lah numbers, we give a generalized identity considering rising and falling factorial powers.

Theorem 3 [*1*] *For any nonnegative integers,* $0 \leq r \leq k \leq n$*, we have*

$$(x+2r)^{\overline{n}} = \sum_{k=0}^{n} \left\lfloor \begin{matrix} n+r \\ k+r \end{matrix} \right\rfloor_r x^{\underline{k}}, \tag{4}$$

where $x^{\underline{k}} = x(x-1)\cdots(x-k+1)$ *and* $x^{\overline{k}} = x(x+1)\cdots(x+k-1)$.

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Scheduling with agreements: new results

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Abstract: In this work, we consider the problem of scheduling with agreements (SWA). This consists in scheduling a set of n jobs non-preemptively on m identical machines subject to constraints that only some specific jobs can be scheduled concurrently. These constraints are given by an agreement graph and the aim is to minimize the makespan. We extend two NP-hardness results for SWA in the case of two machines, processing times at most 3 for bipartite agreement graphs to any class of agreement graphs with a specific structure. We establish complexity results for SWA in the case of split and complement of bipartite graphs. Finally some inapproximability and approximability results are established.

The main results obtained:

Theorem 1: The SWA problem with two machines and processing times in {1, 2, 3} is NP-hard for any class of agreement graphs G=(A U B,E) where A is an independent set with |A|=2a and B is a set of order 2a-b inducing a subgraph with a specific structure. The parameters a and b are two arbitrary integers such that b is less or equal to a.

Theorem 2: The SWA problem for arbitrary split agreement graphs G=(K;S,E), can polynomially be solved in $O(n^3)$ when the processing times of the jobs of K are equal to one and those of S are arbitrary.

Theorem 3: The SWA problem with arbitrary complement of bipartite agreement graphs, unit processing times and release dates in {0, 1, 2} is NP-hard even if m=n.

Theorem 4: The SWA problem is polynomial in the case of complement of bipartite agreement graphs, two distinct release times 0 and r, identical processing times p and m=n.

A q-analogue for bi^{s}nomials coefficients and generalized Fibonacci sequence

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For $s \geq 1$, bi^{s}nomials coefficient denoted by $\binom{n}{k}_s$ are considered as extension of binomial coefficients $\binom{n}{k}$ and are obtained by the multinomial expansion (see [2])

$$\left(1 + x + x^2 + \cdots + x^s\right)^n = \sum_{k \geq 0} \binom{n}{k}_s x^k, \tag{1}$$

Andrew and baxter [1] defined a q-analogue for bi^{s}nomials coefficients by the q-binomials coefficients as follow

For $\alpha = 0, 1, ..., s$

$$\begin{bmatrix} n \\ k \end{bmatrix}_s^{(\alpha)} = \sum_{j_1+j_2+\cdots+j_s=k} \begin{bmatrix} n \\ j_1 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \end{bmatrix} \cdots \begin{bmatrix} j_{s-1} \\ j_s \end{bmatrix} q^{\sum_{r=1}^{s-1}(n-j_r)j_{r+1} - \sum_{r=s-\alpha}^{s-1} j_{r+1}}.$$

Our communication will proceed according to the following steps;

We establish a new expression for the bi^{s}nomials coefficients.

According this expression we define a q-analogue of bi^{s}nomials coefficients $\begin{bmatrix} n \\ k \end{bmatrix}_q^{(s)}$.

With this new definition, we obtain a q-analogue of formula (1).

We suggest a generalized q-Fibonacci sequence which gives for $s = 1$ a Cigler's q-Fibonacci sequence [3].

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Une méthode exacte pour la résolution du problème de l'arbre multi-objectif

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Dans notre étude, nous nous intéressons à la génération de l'ensemble de tous les arbres efficaces d'un graphe connexe $G = (X, E)$ dont chacune des arêtes est munie d'un vecteur de poids de dimension $r \geq 2$, $C = \left(c_{ij}\right)$, pour toute arête e_i de E et $j \in \{1, r\}$. Ce problème est connu pour être de type NP-difficile même pour $r = 2$ [1]. À notre connaissance, peu d'approches de résolution valides ont été implémentées, particulièrement pour des problèmes comportant deux objectifs, [2],[4].

Soit $G = (X, E)$ un graphe connexe, d'ordre n et de taille m, le modèle mathématique associé au problème de l'arbre multi-objectif s'écrit :

$$(P) \begin{cases} Min \left(Z_1 = \sum_{i=1}^{m} C_{i1} x_i , \cdots , Z_r = \sum_{i=1}^{m} C_{ir} x_i \right) & \\ \sum_{i=1}^{m} x_i = n - 1 & (1) \\ \sum_{i=1}^{|E_S|} x_i \leq |S| - 1 \ \ \forall S \subset X, |S| \geq 1 & (2) \\ x_i \in \{0,1\} \ \ \forall i = \overline{1, m} & \end{cases}$$

où $x_i = \begin{cases} 1 \ si \ l'arête \ e_i \ appartient \ à \ un \ arbre \\ 0 \ sinon \end{cases} \quad ; \quad i = \overline{1, m}$

Une méthode exacte est présentée pour le problème de l'arbre multi-objectif. Elle est basée sur un principe de séparation par rapport à des arêtes particulières du graphe, induisant une étape de construction des contraintes du problème, de proche en proche, dans une structure arborescente. Ceci a pour effet de partitionner le graphe initial en sous-graphes de petites dimensions, permettant ainsi d'établir, en chaque feuille *f* de l'arborescence construite, le programme multi-objectif linéaire discret de recherche des arbres efficaces relativement à chaque sous-graphe ($MOST_f$). La résolution du programme $MOST_f$ fait appel à une méthode exacte, par exemple celle décrite dans [3], générant l'ensemble des solutions efficaces relativement à la feuille *f*.

Les résultats de l'expérimentation informatique, en cours de réalisation, sur des instances (*n*,*m*,*r*) pour des valeurs de $n \in \{7, 30\}$, $m \in \{13, 38\}$ et $r \in \{2, 6\}$ sont concluants.

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Identités combinatoires des nombres de r-Lah

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Résumé

Nous établirons des identités combinatoires des nombres de r-Lah en utilisant une approche combinatoire. Une formulation de ces nombres en fonction symétrique est donnée.

1 Introduction

Les nombres de r-Lah, notés $\left\lfloor {n \atop k} \right\rfloor_r$, comptent le nombre de partitions de l'ensemble $\{1, 2, ..., n\}$ en k listes, tel que les nombres $1, 2, ..., r$ sont dans des listes distinctes, voir [2, 1]. Ils satisfont la relation suivante

$$(x+r)^{\underline{n}} = \sum_{k=0}^{n} \left\lfloor {n+r \atop k+r} \right\rfloor_r (x-r)^{\overline{k}}, \tag{1}$$

avec $(x)^{\overline{n}} = x(x+1)\cdots(x+n-1)$ et $(x)^{\underline{n}} = x(x-1)\cdots(x-n+1)$.

les nombres de r-Lah admettent une formule explicite et une relation de récurrence d'ordre deux

$$\left\lfloor {n \atop k} \right\rfloor_r = \frac{(n+r-1)!}{(k+r-1)!}\binom{n-r}{k-r} = \frac{(n-r)!}{(k-r)!}\binom{n+r-1}{k+r-1}. \tag{2}$$

$$\left\lfloor {n \atop k} \right\rfloor_r = \left\lfloor {n-1 \atop k-1} \right\rfloor_r + (n+k-1)\left\lfloor {n-1 \atop k} \right\rfloor_r, \tag{3}$$

tel que $\left\lfloor {n \atop k} \right\rfloor_r = \delta_{n,k}$ pour $k = r$, et δ est le symbole de Kronecker, et $\left\lfloor {n \atop k} \right\rfloor_r = 0$ pour $n < r$.

Théorème 1 *Soient n, k et $r \in \mathbb{N}^3$, on a*

$$\left\lfloor {n+k \atop n} \right\rfloor_r = \sum_{r \le i_1 \le i_2 \le \cdots \le i_k \le n} 2i_1(2i_2+1)\cdots(2i_k+(k-1)). \tag{4}$$

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On Lucas number identities

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Abstract: In this paper, we introduce new identities for Lucas number using some properties of graph coloring.

We denote by G a simple undirected graph of order $n =|V (G)|$ and size $m =|E(G)|$. $N(v)$ is a set of vertices adjacent to vertex v in G and $deg(v)=|N(v)|$ denote its degree. $Link(v)$ denotes a set of edges incident to v. A path P_n, from a vertex v_1 to a vertex v_n, $n \geq 2$, is a sequence of vertices $v_1,\ldots, v_n$ and edges v_iv_{i+1}, for $i = 1,\ldots, n - 1$ and for simplicity we denote it by $v_1,\ldots,v_n$. For a vertex v in G, the graph $G - v$ is gotten from G by removing v and all the edges of G that are incident to vertices of v. The contraction of graph G is the graph G/e obtained by removing e and identifying the vertices u and v incident to e and replacing them with a single vertex v_0 where any edges that were incident on u or v are redirected to v_0.

The well-known Lucas sequence $\{L_n\}$ is defined as $L_0 = 0$, $L_1 = 0$; and $Ln = L_{n-1} + L_{n-2}$, for $n \geq 2$. We call L_n the n -th Lucas number. The Lucas sequence is 0, 0, 1, 1, 2, 3, 5, 8, 13, 21,… Many fields widely applies this sequence, particularly in physics and chemistry [4].

The graph coloring problem is one of the most NP-hard problems [1], it can be described as a mapping that assigns one colour to each vertex in such a way that any two adjacent vertices receive distinct colours, a coloring is optimal if it uses as few colors as possible.

In our communication, we introduce a notion of a graph coloring using by Belbachir and Harik [2] and Dohmen and *al* [3] to involve some identities of Lucas number.

Keywords: Lucas number, graph coloring, cycle.

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Ordonnancement sous contraintes de préparation

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Abstract : In this paper, we consider the problem of scheduling a set of n jobs on two identical machines with preparation constraints. Each job requires before its execution a set of resources and a non-negligible preparation time. The objective is to minimize the makespan. This problem is NP-hard. We prove the NP-hardness of two specific cases where in the first case preparation times take only three values whereas in the second case preparation times and the release dates take only two values, respectively. Then we present some special cases and heuristic algorithms along with an experimental study.

Keywords : Scheduling, identical machines, resources, preparation times, makespan, complexity, heuristics.

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Optimizing over an Integer Efficient Set

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ABSTRACT

The problem of optimizing a linear function over the efficient set of a Multiple Objective Integer Linear Programming (MOILP) problem is known to be difficult, that is, not only for the discrete aspect of the decision variables but also due the unknown structure of the feasible region (the efficient set).

In general, the problem of finding a most preferred efficient point can be written as a mathematical programming problem:

$$(O)\begin{cases} Max\ \varphi(x) \\ s.t. \\ \ x \in Eff \end{cases}$$

where $\varphi : \mathbb{R}^n \to \mathbb{R}$ is a continuous linear function and Eff denote the set of efficient points of MOILP problem mathematically described as:

$$(P)\begin{cases} Max\ Z_i(x) = c^i x; i = 1..r \\ s.t. \\ x \in D = S \cap \mathbb{Z}^n \end{cases}$$

where $r \geq 2$, $S = \{x \in \mathbb{R}^n | Ax \leq b, x \geq 0\}$

Despite its importance in real life applications, a bibliographical research allowed us to conclude that solving (O) have not yet received much attention as it should be. To our knowledge, the only two methods proposed dealing with this topic were given in [1] and [2].

In this work, we propose a new methodology for solving (O) based on a branch and bound technique. Using efficient cuts and tests, our approach succeeds to find the optimal solution for (O) without having to compute all efficient solutions. Furthermore and from experiencing our method on several instances, we have noticed that over the hole set of efficient solutions, just a few ones are found and the algorithm succeeds to find the right optimal solution of program (O). However, as the method explores the domain through efficient cuts, the more the number of constraint is bigger, the more the domain becomes smaller, which allows to the method to go faster. Also, the number of criteria seems to influence our method in a good way. That is, if we raise the criteria's number, our algorithm runs more rapidly.

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Some combinatorial properties of a class of r-Stirling numbers of the first kind

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Abstract. The (singless) Stirling number of the first kind $\left[{n \atop k}\right]$ corresponds to the number of permutations of the set $[n] = \{1, 2, \ldots, n\}$, with exactly k cycles, see for introduction [1] , and the r-Stirling number of the first kind $\left[{n \atop k}\right]_r$ counts the number of permutations of the set $[n] := \{1, 2, \ldots, n\}$ into k cycles such that the first r elements are in distinct cycles. Several authors study these numbers and give them some properties and applications, see for example [2] . In this paper we study a class of r-Stirling numbers of the first kind, named the s-associated r-Stirling numbers of the first kind which count the number of permutations of $[n]$ with k cycles such that the first r elements must be in different cycles with the condition that each cycle is of cardinality at least s. We use combinatorial arguments to give some properties such generating function, congruences, recurrence relations.

Keywords. The r-Stirling numbers of the first kind; generating functions; recurrence relations.

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A class of the Stirling numbers and the values of the generalized Bernoulli polynomials at non-negative integers

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Abstract : The main object of this paper is to introduce and study a class of the Stirling numbers on giving some of their applications on the generalized Bernoulli numbers and polynomials.

Keywords : The associated r-Stirling numbers; the generalized Bernoulli polynomials.

Classification MSC2010 : 11B73; 11B68; 11B83.

1 Introduction

Recall that, the the r-Stirling number of the second kind, $\left\{ {n \atop k} \right\}_r$, counts the number of partitions of the set $[n]$ into k non-empty subsets such that the elements of the set $[r]$ are in different subsets [2], with $\left\{ {n \atop k} \right\}_1 = \left\{ {n \atop k} \right\}_0 := \left\{ {n \atop k} \right\}$ are the Stirling numbers of the second kind.

In a previous paper [5], the authors showed that $B_n^{(\alpha)}(r)$ can be expressed in terms of the r-Stirling numbers of the second kind, where $B_n^{(\alpha)}(x)$ the n-th high order Bernoulli polynomials (see for example [4, 7]). In this paper, we show that the s-aasiociated r-Stirling numbers introduced below are linked to the values at non-negative integers of the generalized Bernoulli polynomials $B_n^{[s-1,\alpha]}(x)$ defined recently by Kurt [3] by

$$\sum_{n\geq 0} B_n^{[s-1,\alpha]}(x)\frac{t^n}{n!} = \left(\frac{t^s}{\exp(t) - \sum\limits_{j=0}^{s-1} \frac{t^j}{j!}} \right)^{\alpha} \exp(xt), \quad s \geq 1.$$

The particular case $\alpha = 1$ was introduced by Natalini and Bernardini [6] (see also [1]).

For $s \geq 1$, we define the s-associated r-Stirling numbers of the second kind by

Definition 1 *The number $\left\{ {n \atop k} \right\}_r^{s\uparrow}$ counts the number of partitions of a n-set into k blocks such that the r first elements are in different blocks and each block from the other $k-r$ blocks is of cardinality $\geq s$.*

From this definition, we assume that

$$\left\{ {n \atop k} \right\}_r^{s\uparrow} = 0, \quad n < sk \text{ or } k < r, \quad \left\{ {0 \atop k} \right\}_r^{s\uparrow} = \delta_{k,0}, \quad n \geq 0 \quad \text{and} \quad \left\{ {n \atop r} \right\}_r^{s\uparrow} = r^{n-r}, \quad n \geq sr.$$

2 Main results

On using combinatorial arguments, the s-associated r-Stirling numbers of the second kind admit an expression given by the following theorem:

Theorem 1 *For $n \geq sk \geq sr \geq 1$, the s-associated r-Stirling numbers of the second kind have exponential generating function*

$$\sum_{n\geq k} \left\{ \begin{matrix} n+r \\ k+r \end{matrix} \right\}_r^{s\uparrow} \frac{t^n}{n!} = \frac{1}{k!} \left(\sum_{i\geq s} \frac{t^i}{i!} \right)^k \exp(rt).$$

Upon using combinatorial arguments, we can derive several three recurrence relations generalized those of the r-Stirling numbers.

An application to the generalized Bernoulli polynomials is given by the following theorems.
For $r,\ m$ be non-negative integers and $\alpha,\ y$ be real numbers, we give below two expressions in terms of the s-associated r-Stirling numbers of the second kind for $B_n^{[s-1,\alpha]}(r)$. The following theorem gives a simplified expression for $B_n^{[s-1,\alpha]}(r)$ for all non-negative integers r.

Theorem 2 *We have*

$$B_n^{[s-1,\alpha]}(r) = \sum_{j=0}^{n} (s!)^{\alpha+j} \frac{n!}{(n+sj)!} \left\{ \begin{matrix} n+r+sj \\ j+r \end{matrix} \right\}_r^{s+1\uparrow} (-\alpha)^{\underline{j}}$$

and for $\alpha = -k$ be a non-positive integer, we have $B_n^{[s-1,-k]}(r) = \frac{n!k!}{(n+sk)!} \left\{ \begin{smallmatrix} n+r+sk \\ k+r \end{smallmatrix} \right\}_r^{s\uparrow}$.

Other expression of the generalized Bernoulli polynomials can be obtained by using the Melzak's formula is given by the following theorem.

Theorem 3 *Let $r,\ n,\ s$ be non-negative integers such that $s \geq 1$. Then, we have*

$$B_n^{[s-1,\alpha]}(r) = \alpha \binom{\alpha+n}{n} \sum_{j=0}^{n} (s!)^{\alpha+j} \binom{n}{j} \frac{(-1)^j}{\alpha+j} \frac{n!j!}{(n+sj)!} \left\{ \begin{matrix} n+r+sj \\ j+r \end{matrix} \right\}_r^{s\uparrow}.$$

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